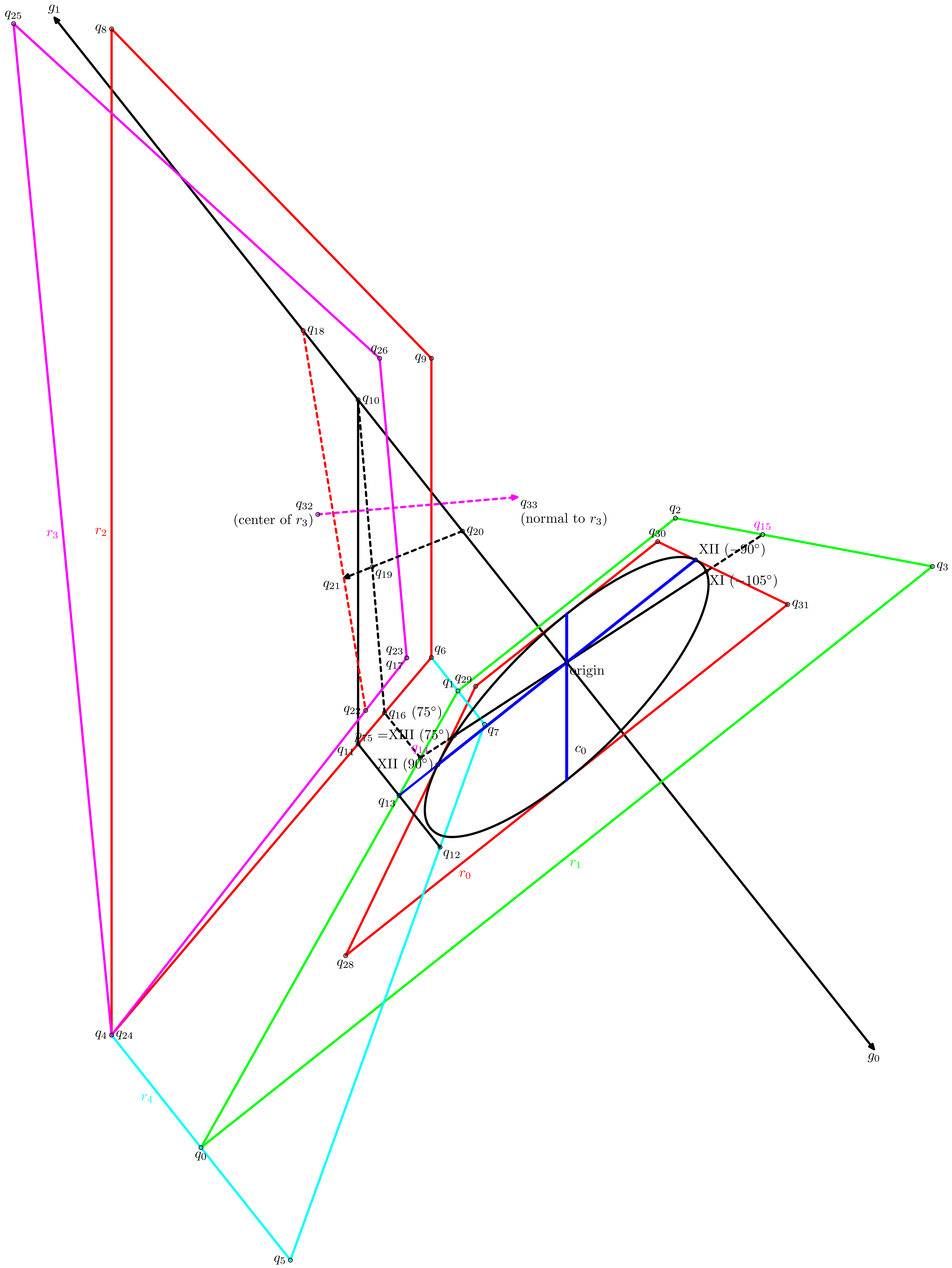
Sundial 1

Laurence D. Finston

Last updated: October 2, 2007



Perspective projection.

See following page for explanation.

 Author: Laurence D. Finston

Let g_0 and g_1 be points on a line passing through the origin such that the line g_0g_1 lies in the x-y plane and its angle to the x-z plane is $51°32'$ (the latitude of Göttingen, Germany). g_0g_1 represents the gnomon.

Let c_0 be a circle with its center at the origin and lying in a plane perpendicular to g_0g_1. Let r_0 be the square enclosing c_0 and r_1 be a larger square in the same plane as r_0 and c_0, whose center is also at the origin and whose sides are parallel to those of r_0.

Let r_4 be a rectangle perpendicular to r_1 such that the vertices q_0 and q_1 of r_1 are the midpoints of the sides q_4q_5 and q_6q_7 of r_4.

Let r_2 be the rectangle $q_4q_6q_9q_8$ such that the vectors $q_8 - q_4$ and $q_9 - q_6$ are vertical, i.e., their y-components are non-zero and their x and z components are 0.

Let q_{13} be the intersection point of the line q_0q_1 with the x-y plane. The line through the origin and q_{13} is the intersection of the x-y plane with the plane of c_0 and represents the projection of the gnomon g_0g_1 onto the plane of c_0 at noon. (The section of this line within the circumference of c_0 is drawn in blue.)

The point q_{10} is the intersection of the gnomon g_0g_1 with the plane of r_2 and the line $q_{10}q_{11}$ is the intersection of the x-y plane with the plane of r_2. It represents the projection of the gnomon g_0g_1 onto the plane of r_2 at noon.

Let point p_{75} be the point on the circumference of c_0 such that the angle between the line from the origin to p_{75} and the line from the origin through q_{13} is $15°$ and the z-coordinate of p_{75} is positive (in a left-handed coordinate system). (The point is to the *right* of the label. This point is also labelled "XIII (75°)".) The line from the origin to p_{75} thus represents the projection of the gnomon g_0g_1 onto the plane of c_0 at 1:00 PM.

The origin and the points q_{10} and p_{75} determine the plane w_0. The point q_{14} is an intersection point of w_0 with the rectangle r_1 and the point q_{16} is an intersection point of w_0 with the rectangle r_2.

The line $q_{10}q_{16}$ thus represents the projection of the gnomon onto the plane of r_2 at 1.00 PM.

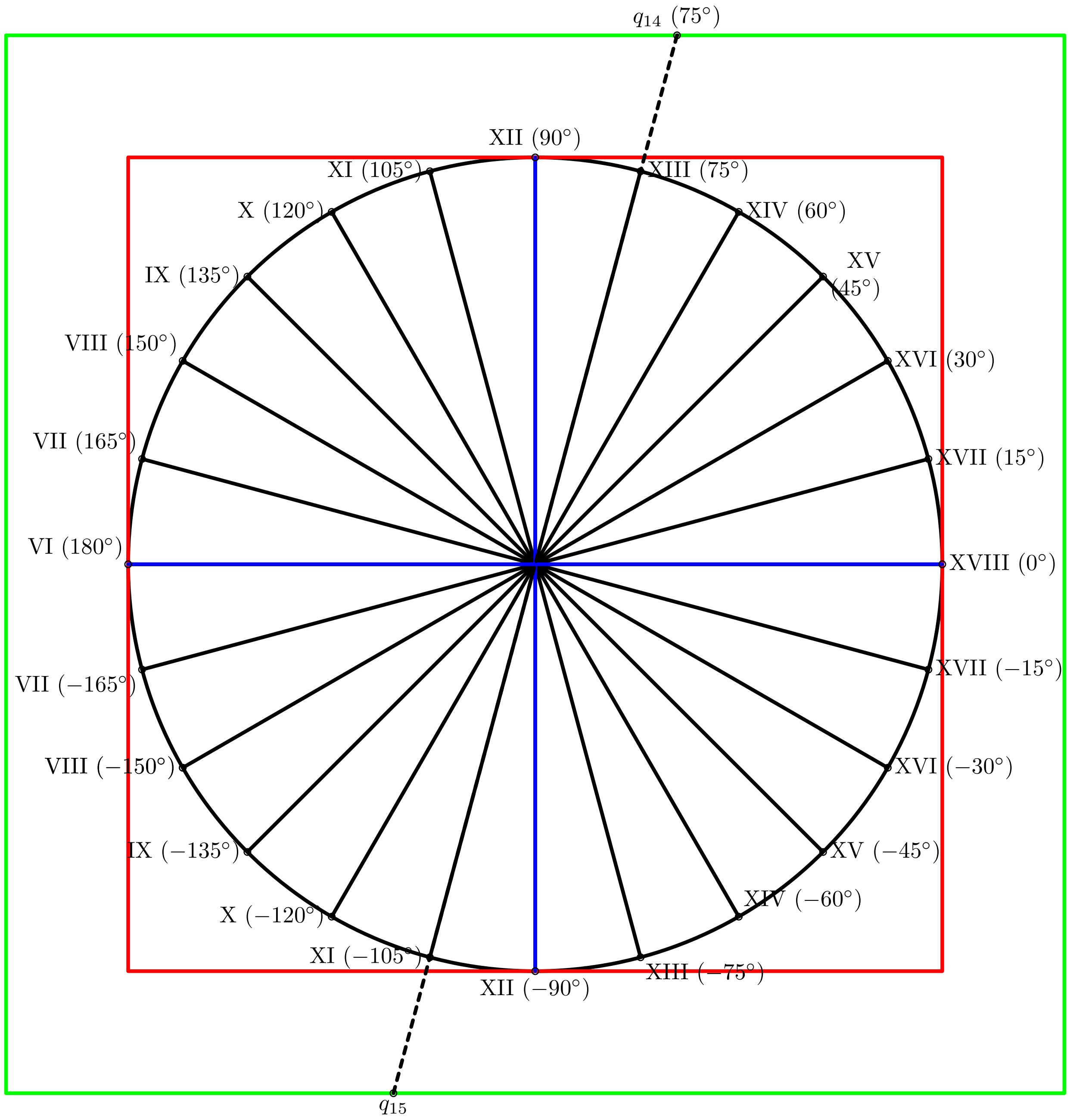
The same principle would apply to any "hour lines" or other lines representing time divisions on c_0, which represents the dial of an equatorial sundial: The intersection of the plane w_n through the origin, a point on the line representing the time division, and a point on the gnomon not in the plane of c_0 and the plane of r_2 will be a line representing the same time division on the plane of r_2. The set of these lines on the plane of r_2 would constitute the dial of a vertical sundial. They would radiate from q_{10}.

In addition, the intersection of a plane w_n representing a time division on c_0 with any other plane v will also represent the corresponding time division on a dial lying in v.

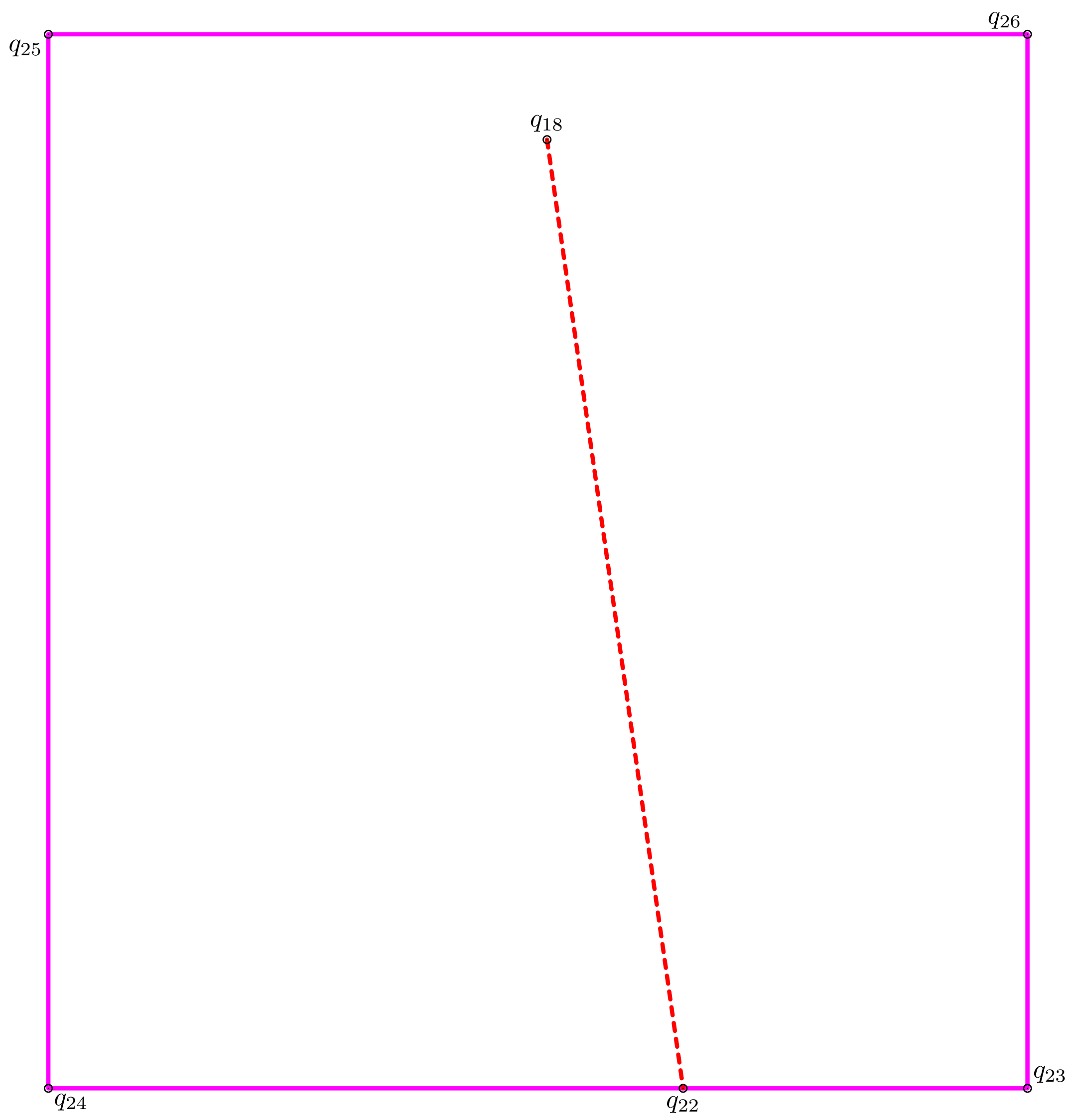
The rectangle r_3 was found by rotating r_2 about the axis q_4q_8 by $5°$ (counterclockwise as seen when looking downward from q_8 onto q_4). The point $q_{17} = q_{23}$ was found by taking the point q_6 and performing the same rotation on it. r_3 was then rotated about the axis q_4q_{17} by $5°$ (counterclockwise as seen when looking from q_4 onto q_17).

The point q_{18} is the intersection of the gnomon g_0g_1 with the plane of r_3. The line $q_{18}q_{22}$ is the intersection of the plane w_0 with the plane of r_3. It thus represents the projection of the gnomon onto the plane of r_3 at 1.00 PM.

 Author: Laurence D. Finston



Parallel projection onto plane of equatorial dial.



Parallel projection onto the skew plane r3.

 Author: Laurence D. Finston